Roll No. .... Total Pages : 6

11781/NJ

D–4/2111

CALCULUS–I

Paper–1101T

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. (a) Using $\in - \delta$ definition prove the limit statement :

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2.$$

(b) Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval [–4, 4]. 10

2. (a) Define continuity of a function at a point. Give an example of a function which is discontinuous at every point of the real interval.

(b) Does the graph of $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & 0 \end{cases}$ has a tangent at origin? Give reasons for your answer. 10

3. (a) Find all the asymptotes of the curve :

$(x - y)^2 (x - 2y) (x - 3y) - 2a(x^3 - y^3) - 2a^2 (x - 2y) (x + y) = 0.$

(b) A hot-air balloon rising straight up from a level field is tracked by range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is 45 degree, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment ? 10

4. (a) Prove that the functions with same derivatives differ by a constant only.

(b) Trace the curve $y = 2x - 3x^{\frac{2}{3}}$. 10

SECTION—B

5. (a) Evaluate : $\int x^{\frac{1}{2}} \sin(x^{\frac{3}{2}} + 1)dx$.

(b) Suppose that f is continuous and that :

$\int_1^2 f(x)dx = 4$. Show that f(x) = 4 atleast once on [1,2]. 10

6. (a) A pyramid 3 m high has a square base that is 3 m on a side. The cross section of pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

(b) Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the x axis. 10

7. Find a power series solution for $y'' + x^2y = 0$. 10

8. (a) Check the convergence of the series :

(i) $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$.

(ii) $\sum_{n=2}^{\infty} \frac{\log n}{\sqrt{n}}$.

(iii) Use partial fractions to find the sum of the series : $\sum_{n=1}^{\infty} \frac{40n}{(2n-1)^2(2n+1)^2}$.

(b) Check for absolute convergence of the series :

$$\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right).$$ 10

SECTION—C

9. Answer the following questions briefly : 3×10=30

(i) What are different types of discontinuities of a function. Define g(4) in a way that extends $g(x)=\frac{x^2-16}{(x^2-3x-4)}$ to be continuous at x = 4.

(ii) Does the curve $y = x^2 - 2x^2 + 2$ have any horizontal tangent? If so, where?

(iii) Define critical point. Does every critical point signals the presence of extreme value? Justify.

(iv) Find the critical points of $y=x^{\frac{5}{3}}-5x^{\frac{2}{3}}$.

(v) The edge of a cube is measured as 10 cm with an error of 1%. The cube's volume is to be calculated from this measurement. Estimate the percentage error in the volume calculation.

(vi) Solve the initial value problem $\frac{dy}{dx}=\frac{1}{x^2}+x,\ x>0;\ y(2)=1.$

(vii) Estimate the average value of $f(x) = x^2$ on the interval [–1,1].

(viii) Express the limit $\lim_{\|P\|\to 0}\sum_{k=1}^{n}(C_k)^2\Delta x_k$, where P is a partition on [0,2].

(ix) State Integral and Ratio test used to check the convergence of a series.

(x) Show that $\sum_{n=1}^{\infty}a_n$ diverges, then $\sum_{n=1}^{\infty}|a_n|$ diverges.